CHAINED MATRIX MULTIPLICATION:

Chain matrix multiplication is an optimization problem that can be solved using dynamic programming. Given a sequence of matrices, the objective is to find out the most efficient way to multiply the matrices. The efficiency parameter is number of multiplications steps.

For example: If matrix A = 12 x 15 and Matrix B = 15 x 25, then after multiplication, resultant matrix will be 12 x 25.
The total number of multiplications required for generating results:

12 x 15 x 25

If it is required to multiply chain of matrices, then number of multiplication steps contributes majorly in deciding efficiency of the process.
The number of multiplication steps can be reduced by selecting the proper sequence of multiplication.

For example: If A = 12 x 20, B = 20 x 14 and C = 14 x 30 then resultant matrix will be 12 x 30.
The possible combinations of multiplications can be: A(BC), (AB)C. But if number of matrices increases, then the sequence selection is difficult task.
Chain matrix multiplication algorithm can be used:

1. To find out minimum number of multiplications required for generating resultant matrix.
2. To find the sequence / combination in which matrices should be used.

Example:
Perform Chain Matrix Multiplication on:

A=13x5 **B=5x89** **C=89x3** **D=3x34**

For above matrices, there are various possible sequences like:
(AB)(CD) A(BC)D A(B(CD)) ((AB)C)D
The best sequence is decided upon number of multiplications required.

Solution:
Step 1: Create an array d[0..n] to store unique dimensions of given matrices.
In the above example, value of n=4, so d[13, 5, 89, 3, 34]
0, 1, 2, 3, 4

Step 2: The number of multiplication required for generating the optimal sequence is decision based on all possible combinations of given matrices and selecting optimal.
To derive all possible combinations, following formulation is used:
A matrix of size m=[n x n] is created. In this matrix all the elements above the diagonal are computed using following equation:

	5785	1530	**2856**
		1335	1845
			9078

For the elements with difference between "i" and "j" is = 1, i.e. i = j-1
M[I,J] = d_{I-1} x d_I x d_{I+1}
For M[1,2] = d0 x d1 x d2 = 13x5x89 = 5785
For M[2,3] = d1 x d2 x d3 = 5x89x3 = 1335
For M[3,4] = d2 x d3 x d4 = 89x3x34 = 9078
For the remaining diagonals:
Formula:

$$M[i,j] = \min[m_{ik} + m_{k+1,j} + d_{i-1} * d_k * d_j]$$
$$\{i \leq k < j\}$$

The value of "k" ranges between "i" and "j"

For M[1,3] → Values of k = 1 and 2

$$M[1,3] = min \begin{matrix} m_{11} + m_{23} + d_0 d_1 d_3 \\ m_{12} + m_{33} + d_0 d_2 d_3 \end{matrix}$$

$$M[1,3] = min \begin{matrix} 0 + 1335 + 13x5x3 \\ 5785 + 0 + 13x89x3 \end{matrix}$$

M[1,3] = min[1530, 9256] = 1530

For M[2,4] → Values of k=2 and 3

$$M[2,4] = min \begin{matrix} m_{22} + m_{34} + d_1 d_2 d_4 \\ m_{23} + m_{44} + d_1 d_3 d_4 \end{matrix}$$

$$M[1,3] = min \begin{matrix} 0 + 9078 + 5x89x34 \\ 1335 + 0 + 5x3x34 \end{matrix}$$

M[1,3] = min[24208, 1845] = 1845

For M[1,4] → Values of k = 1,2,3

$$M[1,4] = \min \begin{matrix} m_{11} + m_{24} + d_0 * d_1 * d_4 \\ m_{12} + m_{34} + d_0 * d_2 * d_4 \\ m_{13} + m_{44} + d_0 * d_3 * d_4 \end{matrix}$$

Replacing values:

$$M[1,4] = \min \begin{matrix} 0 + 1845 + 13 * 5 * 34 \\ 5785 + 9078 + 13 * 89 * 34 \\ 1530 + 0 + 13 * 3 * 34 \end{matrix}$$

M[1,4] = min[4055, 54201, 2856]

From above calculations: the minimum number of multiplications required are : 2856.

The calculations resulting 2856 are referred.

In these calculations: m44 is fixed hence matrix “D” is selected.

The contents of m13 are referred: in which the lowest value is 1335, and matrix m11 is fixed, i.e., “A”. The remaining term in m13 is m23, i.e., {BC}.

Complete multiplication sequence is A {BC} D