# PROBLEM :- 1

Solve the travelling-salesman problem for following matrix using branch and bound method

X=

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | - | 1 | 3 | 4 | 5 |
| 2 | 1 | - | 1 | 4 | 8 |
| 3 | 3 | 1 | - | 5 | 1 |
| 4 | 4 | 4 | 5 | - | 2 |
| 5 | 5 | 8 | 1 | 2 | - |

# SOLUTION

- **Step :- 1**

**a. To find row minimum, subtracting the minimum element from each row**

X=

| | | | | | |
|---|---|---|---|---|---|
| - | 0 | 2 | 3 | 4 | 1 |
| 0 | - | 0 | 3 | 7 | 1 |
| 2 | 0 | - | 4 | 0 | 1 |
| 2 | 2 | 3 | - | 0 | 2 |
| 4 | 7 | 0 | 1 | - | 1 |

**b. Find a column containing all non-zeros entries and subtract minimum element from non-zero column from step-(1a) matrix**

X=

| | | | | |
|---|---|---|---|---|
| - | 0 | 2 | 2 | 4 |
| 0 | - | 0 | 2 | 7 |
| 2 | 0 | - | 3 | 0 |
| 2 | 2 | 3 | - | 0 |
| 4 | 7 | 0 | 0 | - |
| | | | 1 | |

**RESULT :**
**Lower Bound: 1+1+1+2+1+1 = 7**

## Step :- 2

Compute row and column penalty for each zero entries

X=

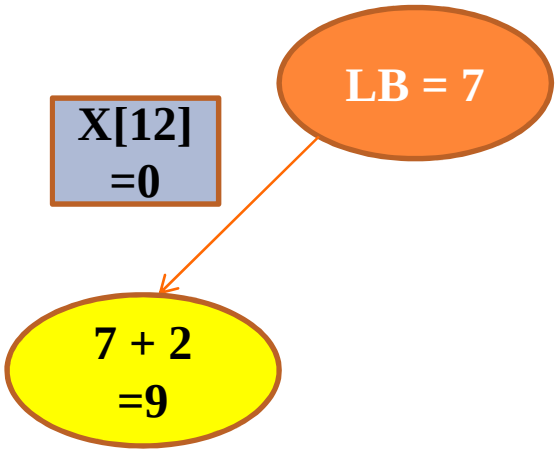
| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | - | 0 [2] | 2 | 2 | 4 |
| 2 | 0 [2] | - | 0 | 2 | 7 |
| 3 | 2 | 0 | - | 3 | 0 |
| 4 | 2 | 2 | 3 | - | 0 [2] |
| 5 | 4 | 7 | 0 | 0 [2] | - |

Maximum Penalty =2

Location = X[1 2] is selected

**If X[1 2] is ignored i.e. X[1 2]=0**

**Then, value 2 is added to lower bound; 7 + 2 = 9**



## Step :- 3

**a. Remove the row and column of maximum penalty value i.e. 1st row and 2nd column and rewrite the matrix**

X=

| | 1 | 3 | 4 | 5 |
|---|---|---|---|---|
| 2 | 0 | 0 | 2 | 7 |
| 3 | 2 | - | 3 | 0 |
| 4 | 2 | 3 | - | 0 |
| 5 | 4 | 0 | 0 | - |

**b. If X[12] is added in tree, then X[21] should not be present on TSP cycle, So make entry X[21]= -**

X=

| | 1 | 3 | 4 | 5 |
|---|---|---|---|---|
| 2 | - | 0 | 2 | 7 |
| 3 | 2 | - | 3 | 0 |
| 4 | 2 | 3 | - | 0 |
| 5 | 4 | 0 | 0 | - |

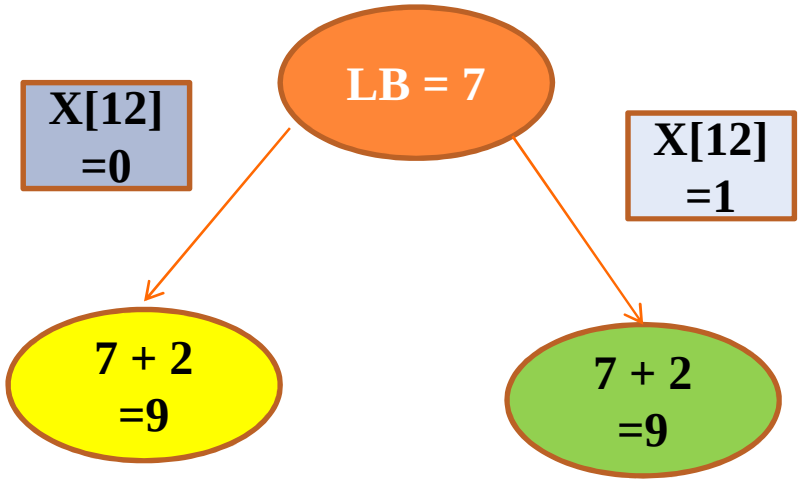
**c. Using from step -(3b) matrix, we have to check every row and every column has a zero; Otherwise subtract minimum element from respective row and column;**
**So 1st column does not have zero; subtract minimum from respective column.**

X=

| | 1 | 3 | 4 | 5 |
|---|---|---|---|---|
| 2 | - | 0 | 2 | 7 |
| 3 | 0 | - | 3 | 0 |
| 4 | 0 | 3 | - | 0 |
| 5 | 4 | 0 | 0 | - |

**2**

**If X[1 2]=1 Then,**
**value at node 7 + 2 = 9**



## Step :- 4

Continue with matrix of step-(3c), compute penalty

X=

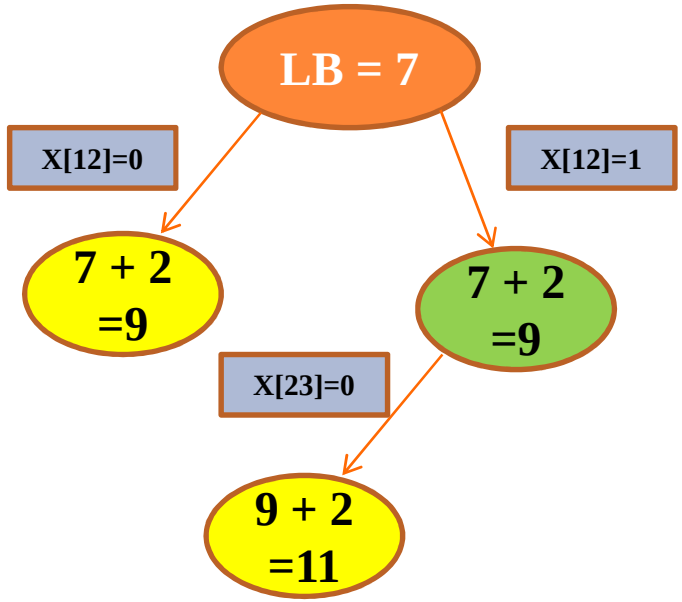
| | 1 | 3 | 4 | 5 |
|---|---|---|---|---|
| 2 | - | 0 [2] | 2 | 7 |
| 3 | 0 | - | 3 | 0 |
| 4 | 0 | 3 | - | 0 |
| 5 | 2 | 0 | 0 [2] | - |

Maximum Penalty = 2

Location = X[2 3]

**If X[2 3] = 0 Then,**

**value 2 is added to lower bound; 9 + 2 = 11**



## Step :- 5

**a. Remove the row and column of maximum penalty value i.e. 2nd row and 3rd column and rewrite the matrix**

X=

| | 1 | 4 | 5 |
|---|---|---|---|
| 3 | 0 | 3 | 0 |
| 4 | 0 | - | 0 |
| 5 | 2 | 0 | - |

**b. If X[2 3] is added, X[3 2] should be removed; it is at present in above matrix.**
**Now, check internal cycles X[1 2]=1 and X[2 3]=1**
**So path is, 1- 2 – 3 , So X[3 1] = -**

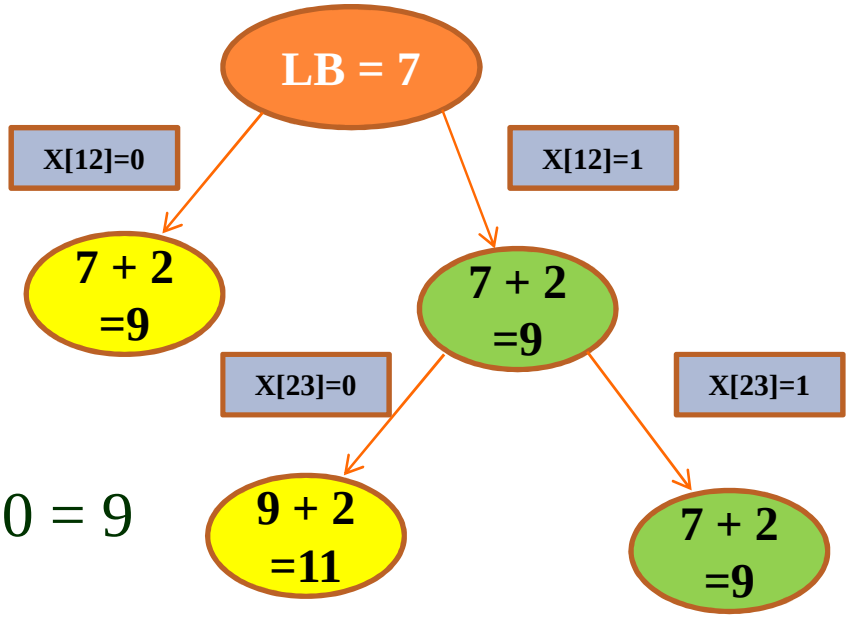
X=

| | 1 | 4 | 5 |
|---|---|---|---|
| 3 | - | 3 | 0 |
| 4 | 0 | - | 0 |
| 5 | 2 | 0 | - |

**c. We have to check every row and every column has a zero; Otherwise add penalty;**

X=

| | 1 | 4 | 5 |
|---|---|---|---|
| 3 | - | 3 | 0 |
| 4 | 0 | - | 0 |
| 5 | 2 | 0 | - |




If X[2 3] = 1 then,

value is remains same i.e. 9 + 0 = 9

## Step :- 6

Continue, Find penalty

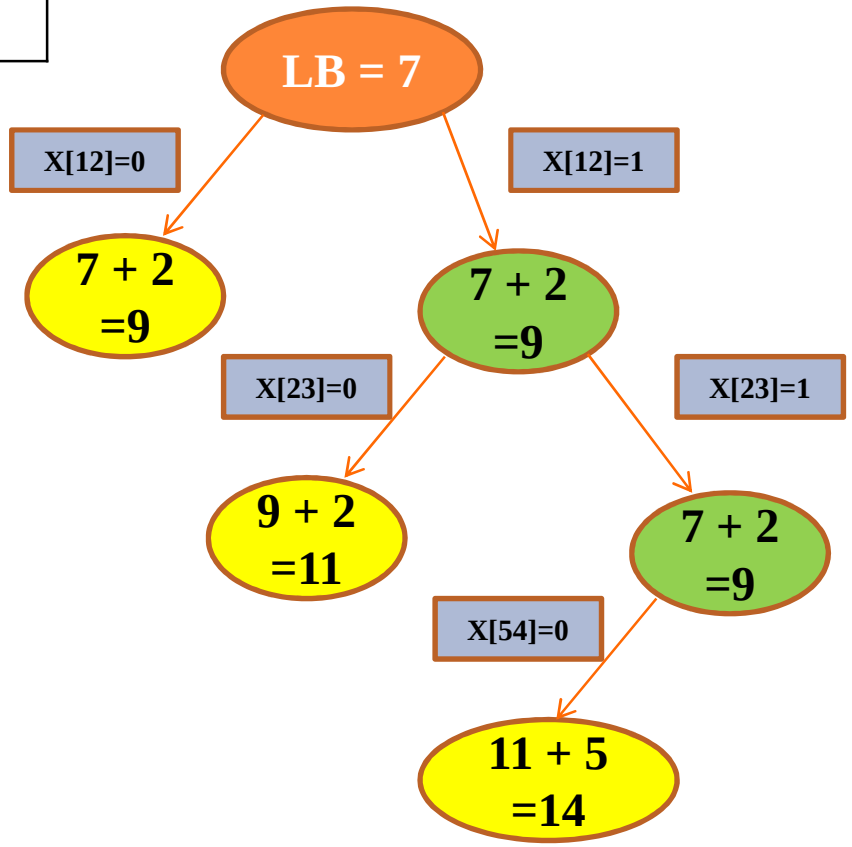
X=

| | 1 | 4 | 5 |
|---|---|---|---|
| 3 | - | 3 | 0 [3] |
| 4 | 0 [2] | - | 0 |
| 5 | 2 | 0 [2+3] | - |

Maximum penalty = 5

Location = X[5 4]

**If X[5 4] = 0 Then, value 5 is added to lower bound; 11 + 5 = 14**



## Remove 5th row and 4th column and rewrite matrix

X=

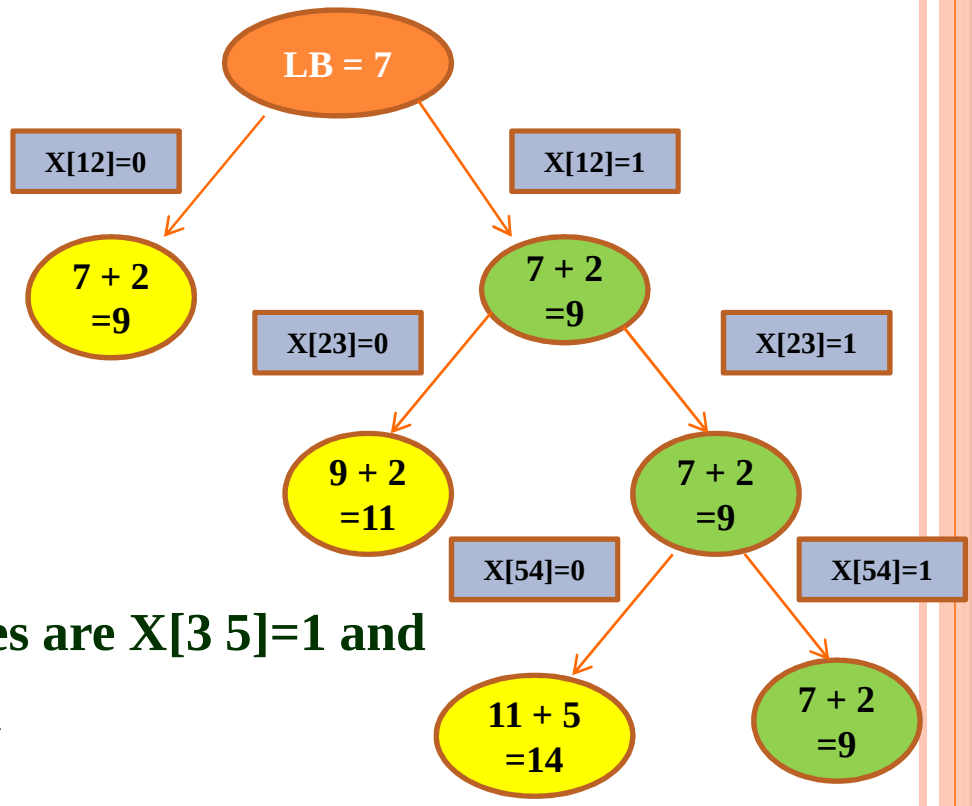
| | 1 | 5 |
|---|---|---|
| 3 | - | 0 |
| 4 | 0 | 0 |

We have every row and column has zero, hence no value is added.

**If X[5 4] = 1 then, value is remains Same i.e. 9 + 0 = 0.**

**Finally,**

**In the above matrix , 2 possible entries are X[3 5]=1 and X[4 1]=1; Consider the entries in path**




**Total Cost Of Path = 9**

**Path : 1 – 2 – 3 – 5 – 4 – 1 = 9**