The Extended Euclidean Algorithm

Example 1: $m = 65, n = 40$

Step 1: The (usual) Euclidean algorithm:

$$
\begin{aligned}
(1) \qquad 65 &= 1 \cdot 40 + \boxed{25} \\
(2) \qquad 40 &= 1 \cdot \boxed{25} + 15 \\
(3) \qquad \boxed{25} &= 1 \cdot 15 + 10 \\
(4) \qquad 15 &= 1 \cdot 10 + 5 \\
10 &= 2 \cdot 5
\end{aligned}
$$

Therefore: $\gcd(65, 40) = 5$.

Step 2: Using the method of back-substitution:

$$
\begin{aligned}
5 &\overset{(4)}{=} 15 - 10 \\
&\overset{(3)}{=} 15 - (25 - 15) && = 2 \cdot 15 - 25 \\
&\overset{(2)}{=} 2(40 - 25) - 25 && = 2 \cdot 40 - 3 \cdot 25 \\
&\overset{(1)}{=} 2 \cdot 40 - 3(65 - 40) && = 5 \cdot 40 - 3 \cdot 65
\end{aligned}
$$

Conclusion: $65(-3) + 40(5) = 5$.

The Extended Euclidean Algorithm

Example 2: $m = 1239, n = 735$

Step 1: The (usual) Euclidean algorithm:

$$
\begin{aligned}
(1) \qquad 1239 &= 1 \cdot 735 + \boxed{504} \\
(2) \qquad 735 &= 1 \cdot \boxed{504} + 231 \\
(3) \qquad \boxed{504} &= 2 \cdot 231 + 42 \\
(4) \qquad 231 &= 5 \cdot 42 + 21 \\
42 &= 2 \cdot 21
\end{aligned}
$$

Therefore: $\gcd(1239, 735) = 21$.

Step 2: Using the method of back-substitution:

$$
\begin{aligned}
21 &\overset{(4)}{=} 231 - 5 \cdot 42 \\
&\overset{(3)}{=} 231 - 5(504 - 2 \cdot 231) && = 11 \cdot 231 - 5 \cdot 504 \\
&\overset{(2)}{=} 11(735 - 504) - 5 \cdot 504 && = 11 \cdot 735 - 16 \cdot 504 \\
&\overset{(1)}{=} 11 \cdot 735 - 16(1239 - 735) && = 27 \cdot 735 - 16 \cdot 1239
\end{aligned}
$$

Conclusion: $1239(-16) + 735(27) = 21$.